

The growth of human settlements during the Neolithic, clustering and food crisis

Sergei Fedotov, David Moss, Daniel Campos

School of Mathematics, The University of Manchester, Manchester, UK

(Dated: April 18, 2008)

We present a stochastic two-population model that describes the migration and growth of semi-sedentary foragers and sedentary farmers along a river valley during the Neolithic transition. The main idea of this paper is that random migration and transition from sedentary to foraging way of life and backward is strongly coupled with the local crop production and the associated degradation of land. We derive a non-linear integral equation for the population density coupled with the equations for the density of soil nutrients and crop production. Our model provides an explanation for the formation of human settlements along a river valley. The numerical results show that the individual farmers have a tendency for aggregation and clustering. We show that the large-scale pattern is a transient phenomenon which eventually disappears due to land degradation.

PACS numbers:

I. INTRODUCTION

The wave of colonization by migrating farmers and establishment of farming communities in Europe between 5000 and 3500 BC is currently a topic of great interest in prehistoric archaeology, linguistics and anthropology [1, 2]. Ammerman and Cavalli-Sforza developed a model for the expansion of farming as a demic diffusion which spread into Europe in the form of wave of advance [3, 4]. Using the radiocarbon dates, they found that farmers spread at an average rate of about one kilometer a year. Interest in simulation and spatial modelling of spread of agriculture has been growing rapidly in the last decade, especially in the physics community [5, 6, 7, 8]. One of the main reasons for this is that the geographical spread of population can be effectively described by the classical Fisher-KPP equation and its various generalizations [9]. These models have attracted considerable interest in physics and biology, because of the huge number of potential applications. Fort and Mendez have applied a time-delayed theory for the Neolithic transition [5] which involves a hyperbolic correction to the Fisher-KPP equation. The transition from hunter-gathering to farming did not happen in a uniform way, and that is why Davison et al. have taken into account both advection and spatial variation in diffusivity and carrying capacity, in the framework of the Fisher-KPP equation [10]. Aoki and Shida [11] have studied the spread of farmers into an area occupied by hunter-gatherers in terms of a system of reaction-diffusion equations. The main objective of those works was to reproduce the observed rate at which agricultural expansion took place in Europe. Despite the interest in establishment of farming communities in Europe, there remains no published material on the spatial structure formed behind the wave of advance. The main challenge of our work presented below is to set up a model which provides an explanation to not only the propagation of farming but also the formation of human settlements. We develop a model that demonstrates the tendency of the distribution of population to form clusters, and furthermore, that this large-scale pattern in population evolution is a transient phenomenon, which disappears due to degradation of land in the form of an extinction wave. Our specific motivation is the successive migration of settlements along parallel river valleys in the Tripolye-Cucuteni system, which has been thoroughly investigated and documented, e.g. [12]. This system can be considered as being one-dimensional, and so is particularly amenable to numerical investigation.

II. A TWO-POPULATION MODEL

A. A two-population model for semi-sedentary foragers and sedentary farmers

In our model the population consists of semi-sedentary foragers and sedentary farmers who share the same territories. The semi-sedentary foragers are the population of individuals randomly moving from place to place along a river valley and searching for food and other resources. An implicit consequence of this behaviour is the foundation of new settlements (large localized values of population density), and an interchange between farming and foraging populations. On the contrary the sedentary farmers are

individuals who do not migrate. They live in small villages scattered near cultivated land in the major river valleys. Their main activities are the cultivation of soil and crop production. In this paper we are interested in the total population density $n(x, t)$ at location x along the river at time t . We define this density as $n = n_1 + n_2$, where n_1 is the density of semi-sedentary foragers and n_2 is the density of sedentary farmers.

We assume that there is not a strict distinction between foraging and sedentary lifestyles. There are always random transitions from a sedentary to a foraging way of life, and *vice versa*; these transitions depend strongly on the local food supply. This is one of the main features of our random walk model. Regarding the movement of semi-sedentary foragers, they do not jump from place to place completely randomly. Unlike Brownian particles in physics, the migration of people cannot be explained by a standard diffusion law, in which the flux is proportional to the gradient of number density of individuals. To describe the random migration of semi-sedentary foragers and random transitions from one lifestyle to another we adopt a biased random walk whose statistical characteristics depend on the local food supply. In our model, the probability of a random migration event making a jump z in the time interval t to $t + \Delta t$, is $\lambda \Delta t$. The probability of transition from foraging lifestyle to the farming is $\alpha_1 \Delta t$. The probability of the conversion of farmers to semi-sedentary foragers is $\alpha_2 \Delta t$. Thus we introduce a new variable, the local crop production per individual per year $q(x, t)$, so that the frequency of jumps λ and transition rates α_1 and α_2 depend on the crop production:

$$\lambda = \lambda(q), \quad \alpha_i = \alpha_i(q) \quad i = 1, 2.$$

It is natural to assume that the frequency $\alpha_2(q)$ is a decreasing function of q . That is, when sedentary farmers are not able to produce enough food to sustain their population, some of them start to migrate from their neighborhood at rate $\lambda(q(x, t))$.

B. Balance equations for population densities

We now set up the balance equation for the density of semi-sedentary foragers, $n_1(x, t)$. According to our random walk model, the density of foragers n_1 at location x at time $t + \Delta t$ can be written as follows

$$n_1(x, t + \Delta t) = (1 - \lambda(q(x, t))\Delta t - \alpha_1(q(x, t))\Delta t) n_1(x, t) + \int \lambda(q(x - z, t))\Delta t n_1(x - z, t) \rho(z) dz + \alpha_2(q(x, t))\Delta t n_2(x, t). \quad (1)$$

This equation is a conservation law for foragers. The first term on the right hand side represents those foragers who stay at location x and do not move during time Δt and do not become sedentary farmers. The second term gives the number of foragers who arrive at x during time Δt from different places $x - z$, where the jump distance z is distributed by dispersal kernel $\rho(z)$. The last term is the number of sedentary farmers who become the semi-sedentary foragers during the time Δt .

The sedentary farmers do not migrate, and their density $n_2(x, t)$ obeys the balance equation involving logistic growth and lifestyle transitions:

$$n_2(x, t + \Delta t) = (1 - \alpha_2(q(x, t))\Delta t) n_2(x, t) + r n_2(x, t) \left(1 - \frac{n(x, t)}{K(q)}\right) \Delta t + \alpha_1(q(x, t))\Delta t n_1(x, t), \quad (2)$$

where r is the growth rate of the sedentary population. The carrying capacity K is in general, an increasing function of the local crop production q . The last term gives the number of foragers who become the farmers during the time Δt . In the limit $\Delta t \rightarrow 0$, from (1) and (2) we obtain two differential equations for $n_1(x, t)$ and $n_2(x, t)$:

$$\frac{\partial n_1}{\partial t} = \int \lambda(q(x - z, t)) n_1(x - z, t) \rho(z) dz - \lambda(q) n_1 - \alpha_1(q) n_1 + \alpha_2(q) n_2 \quad (3)$$

$$\frac{\partial n_2}{\partial t} = r n_2 \left(1 - \frac{n}{K(q)}\right) + \alpha_1(q) n_1 + \alpha_2(q) n_2. \quad (4)$$

In this paper we are interested in the evolution of population density $n(x, t)$ on characteristic time scales around 100 – 500 years. Therefore we can adopt a local equilibrium for the two populations, describing the balance between the nomadic and sedentary ways of life in proportions p and $1 - p$. We write

$$n_1(x, t) = pn(x, t), \quad n_2(x, t) = (1 - p)n(x, t), \quad (5)$$

where the proportion of foragers p is the function of the crop production q :

$$p = p(q). \quad (6)$$

In the asymptotic regime when $\alpha_i \gg \lambda$, one can write the dependence of p on the transition rates $\alpha_1(q)$ and $\alpha_2(q)$ as

$$p(q) = \frac{\alpha_2(q)}{\alpha_1(q) + \alpha_2(q)}. \quad (7)$$

The evolution equation for biased migration (3) and population growth (4) can be rewritten as one balance equation for the total population density. By adding equations (3) and (4) and using (5) we obtain a single equation for $n(x, t)$

$$\begin{aligned} \frac{\partial n}{\partial t} = & \int p(q(x - z, t))\lambda(q(x - z, t))n(x - z, t)\rho(z)dz \\ & - p(q(x, t))\lambda(q(x, t))n(x, t) + (1 - p(q))rn \left(1 - \frac{n}{K(q)}\right). \end{aligned} \quad (8)$$

Now we need to find an approximate form for the function $p(q)$. It is natural to assume that for a large crop production q , the population mostly consists of sedentary farmers, that is, p close to zero, while for a low crop production the population mostly is semi-sedentary foragers (p is close to one). A simple approximation for p might be

$$p(q(x, t)) = \mathcal{H}(q(x, t) - q_{\min}),$$

where $\mathcal{H}(x)$ is the Heaviside step function: $\mathcal{H}(x) = 0$ if $x > 0$; $\mathcal{H}(x) = 1$ if $x \leq 0$. Thus farmers do not migrate if the crop supply is greater than a critical value q_{\min} . In our numerical simulations we use a piece-wise approximation: a linearly decreasing function in the interval between the minimum value q_{\min} and the maximum value q_{\max}

$$p(q(x, t)) = \begin{cases} p_{\max}, & q \leq q_{\min}, \\ -aq + b, & q_{\min} < q < q_{\max}, \\ p_{\min}, & q \geq q_{\max} \end{cases}.$$

Based on [13], we take $q_{\min}, q_{\max} = 300, 736$ kg per person per year respectively. For the results discussed below we set $p_{\max} = 0.95$, $p_{\min} = 0.05$ (see Fig. 1) but the exact values are not crucial to our results.

The dispersal kernel $\rho(z)$ is assumed to have two cutoffs: a minimum cutoff (foragers do not travel very small distances) and a maximum cutoff (foragers do not migrate over very large distances in a single jump). In numerical simulations we use minimum cutoff Δz of between 5 and 20 km. The maximum cutoff is more difficult to estimate. However, since the typical length of the river is around 500 km, it seems reasonable to assume that the maximum cutoff is around 50 km (see Fig. 1).

C. Equation for crop production

We assume that the human population derives most of their food from the cultivation of land. Thus we suggest the following formula for the local food production $q(x, t)$

$$q(x, t) = \alpha \left(\frac{n(x, t)}{n_0 + n(x, t)} \right) \left(1 - e^{-\beta F(x, t)} \right), \quad (9)$$

where $F(x, t)$ is the density of soil nutrients, α is the production rate coefficient, and β is the parameter that determines how the yield depends on the nutrients. This equation describes how the rate of food

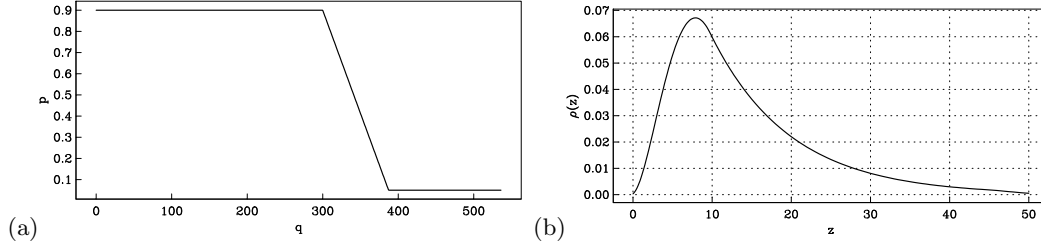


Fig. 1: (a) The function $q(p)$ and (b) $\rho(z)$ for $\Delta z = 10$ km.

supply q increases due to the increase in the population density n , and how the degradation of land (the decrease of soil nutrients F) leads to a decrease of food production through the factor $1 - e^{-\beta F(x,t)}$. Note that the factor $\frac{n(x,t)}{n_0 + n(x,t)}$ describes a tendency toward group solidarity that increases the efficiency of food production. We know from existing data that the yield, i.e. the production of food per unit area, can be up to 0.0736 kg/m²year [13, 14]. We estimate that the area cultivated by people is approximately 10⁴ m² per person. Thus, we have an estimate for α of 736 kg per person per year. In general, it is difficult to model how the production of food depends on the nutrients, since this relation is strongly dependent on the environmental conditions. One of the existing models is given by the Mitscherlich-Baule yield response [15] which takes into account productivity losses due to soil erosion. Following this model, in [16] the factor $1 - e^{-\beta F(x,t)}$ has been proposed, and this is what we have introduced into our model. The value $\beta \simeq 890$ m²kg⁻¹ is found for the corn cultivation. However, other studies [17] seem to suggest lower values ($\beta \simeq 65$ m²kg⁻¹).

D. The land degradation equation

We adopt the following model for nutrient depletion and the corresponding land degradation. We assume that equation for the density of soil nutrients $F(x,t)$ has the form

$$\frac{\partial F(x,t)}{\partial t} = \xi_1 - \xi_2 F(x,t) - \gamma q(x,t)n(x,t) \quad (10)$$

where ξ_1 is the rate at which soil nutrients regenerate naturally, ξ_2 is the rate of nutrient depletion due to environmental reasons (erosion, flooding, etc.), γ is the rate at which nutrients are depleted due to the harvests. Here we assume that natural nutrient depletion is much slower than depletion due to human activity. In order to estimate γ , we can use the data from [14]. This data corresponds to prehistoric agriculture in Hawaii; however, similar values are cited for Sub-Saharan agroecosystems and there is no data available (as far as we know) for other regions. So using that study we estimate $4 \cdot 10^{-4}$ kg_P/m²yr, where kg_P denotes kilograms of phosphorus in the soil. This value, together with the data above of 0.0736 kg/m²yr lead us to assume that the rate of nutrient depletion $\gamma = 5.43 \cdot 10^{-3}$ kg_P/kg. However, other studies mention that the total concentration of nutrients in crops can be up to 3% [17]. It would suggest that the value $\gamma = 0.03$ kg_P/kg can be taken as a maximum threshold. Of course, other nutrients are also important, but phosphorous can be regarded as a proxy for them. Soil regeneration is a very complex process and depends on many environmental parameters. Nevertheless, in some agroecconomics models regeneration is modeled as a constant [18] as we have assumed in our model.

III. NUMERICAL RESULTS

Numerical simulations of the non-linear integral equation (8) together with (9) and (10) reveal a very interesting dynamical behaviour: the emergence of large-scale patterns in population density. This phenomenon can be interpreted as the formation of human settlements along a river valley. Thus our model provides an explanation for formation of settlements as a dynamical phenomenon. The individual farmers have a tendency for aggregation and clustering as a result of non-linear random migration. Moreover our model describes not just a population clustering but subsequent decay of these clusters (settlements) due to land degradation.

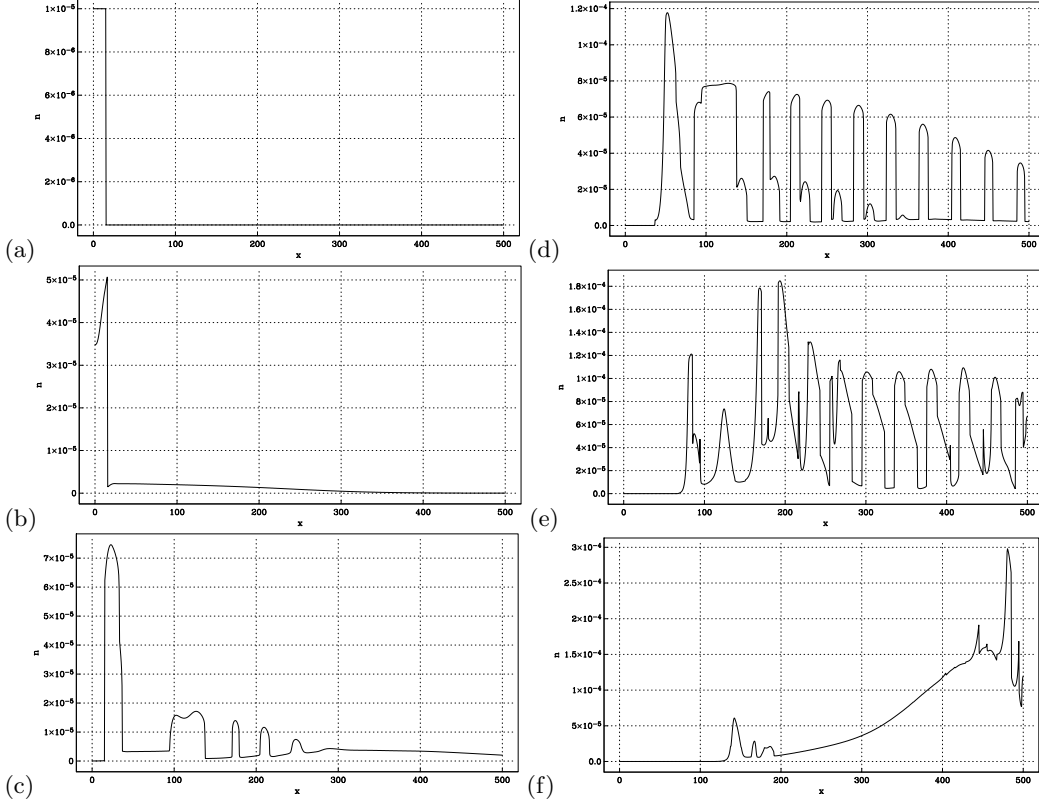


Fig. 2: $n(x, t)$ for the model with $\lambda = 0.2$, $\gamma = 0.01$, $\Delta x = 10$ discussed in the text. The panels (a) – (f) show successively the solution at times 0 – 500 yr, at intervals of 100 yr.

Our model has a rather large number of parameters, but fortunately most of them can be estimated from existing sources, as indicated above. For numerical work we use as unit of time 1 yr, measure densities in units of m^{-2} , and assume $\lambda = \text{const}$. We take the values $r = 0.03$, $\alpha = 736$, $\beta = 200$, $K = 10^{-4}$, $\xi_1 = 10^{-4}$, $\xi_2 = 0$, $q_{\min} = 300$, $q_{\max} = 736$. Note that for simplicity in this initial study we take K to be a constant, rather than a function of q . We explore ranges $0.01 \leq \gamma \leq 0.03 \text{ yr}^{-1}$, $0.05 \leq \lambda \leq 0.3 \text{ yr}^{-1}$, $5 \leq \Delta z \leq 20 \text{ km}$. Our initial conditions were rather arbitrary: $n = 0.1K$ in $0 \leq x \leq 15 \text{ km}$, $n = 0$ elsewhere, although from experiments with other initial conditions with $n(x, 0)$ non-zero in $x \lesssim 100$ it appears that results are quite insensitive to details of the initial state. Again for simplicity, we set the initial value $F(x, 0)$ to be uniform and equal to F_0 . Our ‘standard’ choice was $F_0 = 0.01$. If the range of x were to be extended to negative values, the evolution of $n(x, t)$ would be approximately symmetric about $x = 0$.

Fig. 2 shows the population density $n(x, t)$ for $\lambda = 0.1$, with $\Delta x = 10$, $\gamma = 0.01$, at intervals of 100 yr. Panel (a) shows the “arrival” of farmers at location $0 < x < 15$ along the river. After 300 years, one can see clear evidence of clustering of population (“settlements”). Panel (e) shows their subsequent decay and reappearance of clusters behind the extinction wave. The entire population decays over about 800 yr, depending to some extent on the exact choice of parameters. A general feature of our modelling is that most population clusters grow and decay *in situ*, without significant movement. We can make an order of magnitude estimate of the total population of a cluster by taking the linear extent of a cluster as the diameter of a circular settlement. This gives typical figures of $O(10^4)$ individuals.

We also explored the dependence of the speed of population advance, and the separation of the clusters, as a function of the parameters. The speed of advance is rather ill-defined, but for simplicity we monitored the movement of the main peak of the distribution. (Arguably we could have, e.g., studied the position of the cluster furthest from the origin.) We found that the speed of advance depends approximately linearly on both λ and γ , and to be insensitive to Δx . The separation of clusters is independent of both λ and γ and depends approximately linearly on Δx ($= (1.5 - 2)\Delta x$). If $\lambda \lesssim 0.05$ or $\Delta x \gtrsim 15$ the clustering phenomenon does not occur. While we have concentrated our efforts on studying variation among a manageable subset of parameters while retaining fixed plausible values for the others, we did also make

a small study of the effects of varying F_0 and ξ_1 . Increasing the value of F_0 , up to 0.1 from our canonical value of 0.01, prolongs the timescale of the population evolution, by a factor of up to about 2. Values of F_0 significantly smaller than 0.01 do not give clustering for typical choices of the other parameters. It means that for non-fertile soils transition to farming and the emergence of clusters are unlikely to occur. For larger values of ξ_1 , $\gtrsim 10^{-4}$, the regeneration is so strong that, although clustering occurs initially much as for $\xi_1 = 10^{-4}$, the final state is a spatially uniform population. For smaller values of ξ_1 , e.g. $\xi_1 = 10^{-5}$, we still see strong clustering, but the overall phenomenon persists for a significantly shorter time, as the episodes of cluster regeneration seen when $\xi_1 = 10^{-4}$ do not now occur.

IV. DISCUSSION AND CONCLUSIONS

We have explored a continuous model of linear (one-dimensional) population migration and clustering. We took our inspiration and guidance from the successive south to north migration of the Tripolye-Cucuteni cultures along parallel river valleys [12]. We attempted only to illustrate the migration along a single valley. The main result is that our model gives an explanation for the formation of settlements as a dynamical phenomenon. The individuals have a tendency for aggregation and clustering as a result of non-linear random migration. Moreover our model describes subsequent decay of these clusters (settlements) due to land degradation in the form of an extinction wave. Clearly our model, being mathematically continuous, cannot be expected to reproduce in detail the essentially discrete phenomenon of the establishment, growth and decay of major settlements and their satellites, as revealed by the archaeological record – our goal is far more modest. For reasonable choices of parameters we obtain migration over distances of order 500 km in times of 500 – 1000 yr. Without fine tuning, our solutions exhibit distinct clustering of population, at intervals of 10 – 30 km. Plausibly these clusters are estimated to have $O(10^4)$ individuals. These estimates are all consistent with what is known about the Tripolye-Cucuteni cultures [13]. Overall, we feel that our quite naive modelling captures some of the essence of the clustering seen in population migration, and points the way for more sophisticated modelling. We can think of a number of significant developments in the future, including taking $K = K(q)$ and allowing for a latency effect, in which population in a cluster (‘settlement’) has a reduced probability of moving, representing an attachment to the investment of building a house and clearing land. It is easy to allow for additional resources available from e.g. forest or river, which are maybe harder to exhaust than the fertility of the land and so can act as a reservoir of resource. It would be interesting to apply our model to understand societal collapses to which environmental problems as habitat destruction, soil degradation and overpopulation contribute [19].

Acknowledgment

The research is supported by the European Community’s Sixth Framework Programme, grant NEST-028192-FEPRE.

-
- [1] V. G. Childe, The dawn of European civilisation. (London: Routledge and Kegan Paul; 1968).
 - [2] T. D. Price (Ed.), Europe’s first farmers (Cambridge University Press, Cambridge, 2000).
 - [3] A. J. Ammerman, and L. L. Cavalli-Sforza, *Man* **6**, 674 (1971).
 - [4] A. J. Ammerman, and L. L. Cavalli-Sforza, (1984) *The Neolithic Transition and the Genetics of Populations in Europe* (Princeton Univ Press, Princeton, 1984).
 - [5] J. Fort, and V. Mendez, *Phys Rev Lett* **82**, 867 (1999); *Phys. Rev. E* **60**, 5894 (1999).
 - [6] M. O. Vlad, and J. Ross, *Phys. Rev. E* **66**, 061908 (2002).
 - [7] J. Fort, T. Pujol, L. L. Cavalli-Sforza. *Camb Archaeol J.* **14**, 53 (2004).
 - [8] J. Fort, J. Pérez-Losada, N. Isern. *Phys Rev E* **76** 031913 (2007).
 - [9] U. Ebert and W. van Saarloos, *Physica D* **146**, 1 (2000); *Phys. Rep.* **337**, 139 (2000).
 - [10] K. Davison, P. Dolukhanov, G. R. Sarson, and A. Shukurov *J Arch Sci* **33**, 641 (2006).
 - [11] K. Aoki, S. Mitsuo and S. Nanako, *Theor. Population Biology*, **50**, 1 (1996).
 - [12] M. Zvelebil and P. Dolukhanov, *J. of World Prehistory* **5**, 233 (1991).
 - [13] P. Dolukhanov (private communication)

- [14] A. S. Hartshorn, O. A. Chadwick, P. M. Vitousek and P. V. Kirch. Proc. Natl. Acad. Sci. USA **103**, 11092 (2006).
- [15] H. D. Patterson, Biometrics, **25**, 159 (1969).
- [16] J. W. Hopkins, R. Lal, K. D. Wiebe and L. G. Tweeten. Land Degrad. Develop. **12**, 305 (2001).
- [17] A. R. Overman, R. V. Scholtz III and F. G. Martin. Commun. Soil Sci. Plan. **34**, 851 (2003).
- [18] K. E. McConnell. An Economic Model of Soil Conservation. Am. J. Agric. Econ. **65**, 83 (1983).
- [19] J. Diamond, Collapse: How Societies Choose to Fail or Succeed (Penguin books, London, 2005).